Propulsive Efficiency from an Energy Utilization Standpoint

John Hiram Lewis III*
Hartford Graduate Center, Rensselaer Polytechnic Institute, Hartford, Conn.

The standard but ill-defined concept of propulsive efficiency, classically employed in the analysis of turbojet and turbofan engines, can be approached from the point of view of energy utilization by applying basic thermodynamic principles. First and second law analysis of propulsive powerplants as general thermodynamic systems leads to a universal definition for propulsive efficiency. This definition clearly accounts for the energy unavailability production separated into two distinct parts: 1) the unavailable energy associated with the thermodynamic cycle's rejected heat; and 2) the wasted energy produced by inefficiencies inherent to the conversion of available cycle energy to propulsive power. As a result, conversion or transfer inefficiencies are consistently reflected in the propulsive efficiency, whereas cycle inefficiencies are most properly identified with cycle, or thermal, efficiency for a powerplant.

Nomenclature

E_{diss} = dissipated energy quantity
F_n = engine net thrust
g = gravitational constant
h = specific enthalpy
HV = equivalent fuel (lower) heating value
J = unit conversion constant (Ft. Lb./BTU)
KE = kinetic energy
Q_A = heat quantity added
Q_{diss} = heat quantity dissipated
Q_{rej}, Q_r = heat quantity rejected
S = entropy
V_D = fan bypass stream exhaust velocity
V_e = engine (primary) exhaust velocity
V_j = turbojet jet velocity
V_0 = freestream, or flight velocity
W_D = fan bypass mass flow rate (duct flow)
W_e = engine (primary) exhaust mass flow rate
W_f = fuel mass flow rate
η = efficiency

Introduction

OFTEN, a return to basic fundamentals provides essential insight to help the powerplant performance analyst understand and analyze new aircraft powerplant concepts. This usually means evaluating an engine's overall efficiency in terms of propulsive and cycle, or thermal efficiencies. However, there has been some confusion as to the proper definition of these interrelated terms. Standard textbooks in the gas turbine field present definitions for propulsive efficiency which tend to reflect earlier turbojet analysis.[1-3] As a result, these definitions are not sufficiently general to cover theoretical analysis of the currently more prevalent turbofan engines. The following discussion is intended to help clarify some of this confusion by providing and explaining a single, propulsive efficiency definition that consistently applies to all circumstances. The basic conceptual foundation upon which this definition rests relates to the changes in form of energy in the purest thermodynamic sense. With this in mind, the actual propulsive efficiency definition is preceeded by a discussion of the First and Second Laws of Thermodynamics as applied to a propulsion system. In these days of increased emphasis on fuel conservation, an approach to propulsion analysis which focuses on energy as the conceptual foundation can provide particularly useful insight for the engine design and analysis process.

Received Jan. 6, 1975; revision received May 27, 1975.
Index category: Airbreathing Propulsion, Subsonic and Supersonic.

*Adjunct Associate Professor of Mechanical Engineering.

Conservation of Energy

Consider the application of the First Law of Thermodynamics (energy balance) to a propulsion system moving at velocity V_0. The stationary observer would see chemical energy added to the system by the fuel† and propulsive work done by the propulsion system on the traveling vehicle. He would also see residual energy being dissipated to the surrounding atmosphere from the exhaust gases left behind by the propulsion system. This dissipated energy would be observed in the hot exhaust gases in two separately identifiable forms: 1) residual exhaust kinetic energy; and 2) residual thermal energy. (In the case of a separate flow turbofan, the two forms of dissipated energy exist in each exhaust stream.)

In terms of time rates of change, the energy balance for the total system takes the form:

rate of (chemical) energy addition = propulsive power + rate of energy dissipation (1)

where,

dissipated energy = residual kinetic energy + residual exhaust gas thermal energy

In symbol form (see Fig. 1):

energy addition rate $= W_f \times \text{HV} \times J$
propulsive power $= F_n \times V_0$
residual kinetic energy rate $= \frac{1}{2}g(V_e - V_0)^2 \times W_e + \frac{1}{2}g(V_D - V_0)^2 \times W_D$
residual exhaust gas heat rate $= Q_{diss}$

Thus, the energy balance takes the form‡

$$W_f \times \text{HV} \times J = F_n \times V_0 + \tfrac{1}{2}g(V_e - V_0)^2 \times W_e + \tfrac{1}{2}g(V_D - V_0)^2 \times W_D + Q_{diss} \quad (1)$$

Energy Unavailability

According to the consequences of the Second Law of Thermodynamics, the net dissipated thermal energy Q_{diss} can be interpreted as a summation of individual constituent components. These components are illustrated in *h-s* diagrams (Fig. 2), which trace out as process paths state properties of the flow traversing through the propulsive system. Individual

†Most precisely, the total fuel energy also includes intrinsic and kinetic energy quantities, which are usually sufficiently small as to be neglected.
‡Units: ft-lbf/sec.

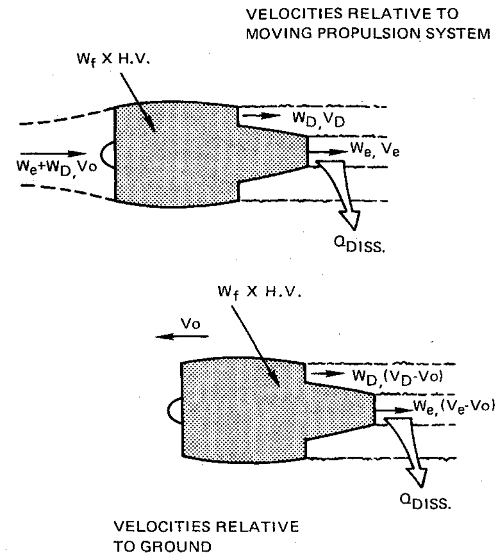

Fig. 1 Exhaust residual energy in a two-stream turbofan engine.

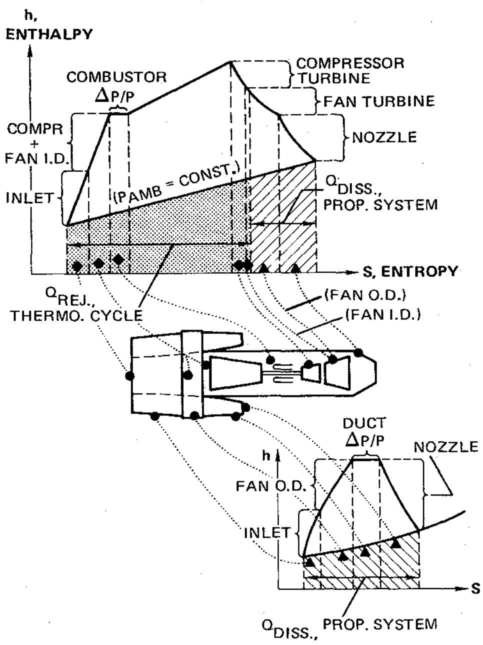

Fig. 2 Thermodynamic processes in a conventional turbofan.

quantities of unavailable energy are represented on these diagrams as sections of areas of $\int Tds$ under the ambient pressure line. In the engine exhaust, energy unavailability is the consequence of a combination of component inefficiencies (including inlet and nozzle) and combustor heat addition. Ultimately, any entropy-producing process of the engine acting on the air will manifest itself as excess thermal energy appearing in the engine exhaust gases. If the thermodynamic cycle is to be associated with overall cycle pressure ratio (which includes the ram pressure ratio) and maximum turbine temperature, then the sum total of unavailable energy created by all processes related to the realization of pressure ratio or turbine temperature should be considered to represent the heat rejection ($Q_{\rm rej}$) normally attributed to the thermodynamic cycle. The balance of the dissipated energy is directly associated with the production of thrust by the system as a propulsive device. With this point of view, the dissipated exhaust energy can be divided into two components

$$Q_{\rm diss} = Q_{{\rm rej}_{\rm (thermo\ cycle)}} + Q_{{\rm diss}_{\rm (propulsive\ system)}} \tag{2}$$

For each component, the constituent contribution to dissipated energy from each process is illustrated on the *h-s* diagram of Fig. 2. For example, entropy production chargeable to the thermodynamic cycle would occur in turbines, compressors, and the gas generator flowpath pressure losses; entropy production chargeable to the propulsive system would occur in nozzles, fans, the fan-drive turbine, and fan duct pressure losses.

Propulsion Analysis

The energy balance of Eq. (1) can be rewritten in the following form, as suggested by Eq. (2)

$$Q_{\rm added} - Q_{\rm rej} = F_n \times V_0 + \text{residual KE} + Q_{\rm diss}\text{,prop. sys.} \tag{3}$$

or, more generally

$$Q_A - Q_R = F_n \times V_0 + E_{\rm diss} \tag{4}$$

where

$E_{\rm diss}$ = residual KE + $Q_{\rm diss}$ = net wasted energy due to thrust production

If, ideally, $E_{\rm diss} = 0$, that is all losses associated with the production of thrust are assumed to vanish, then the term $(Q_A - Q_R)$ can be seen to be identical to the maximum possible (ideal) propulsive power. It is also exactly equal to the net thermodynamic cycle work. This concept of ideal propulsive power naturally leads to the following definition for propulsive efficiency

$$\eta_{\rm prop} = \frac{\text{actual propulsive power}}{\text{maximum available (ideal) propulsive power}} \tag{5}$$

where

Ideal propulsive power = actual propulsive power
+ energy dissipation due to thrust production
= net cycle output energy

Maximum available propulsive power often is simply referred to as the energy available for propulsion.

The use of the quite universal definition for propulsive efficiency in Eq. (5) always will lead to a consistent understanding of propulsive effects on overall engine efficiency, as distinguished from thermodynamic cycle effects. A companion definition to Eq. (5) would be

$$\eta_{\rm thermal} = \frac{\text{maximum propulsive power available}}{\text{thermal energy added}} \tag{6}$$

and

$$\eta_{\rm overall} = \frac{\text{actual propulsive power}}{\text{thermal energy added}} \tag{7}$$

which means that

$$\eta_{\rm overall} = \eta_{\rm prop} \times \eta_{\rm thermal} \tag{8}$$

Thermal efficiency, as defined by Eq. (6), can be related directly to cycle efficiency in the classic thermodynamic sense. However, these two efficiencies are not identical. The heat added to the air is usually somewhat less than the available fuel chemical energy due to combustion process effects.

It should be emphasized that the universal propulsive efficiency definition accounts for wasted kinetic energy and for thermal energy dissipation. The thermal dissipation component is often collectively referred to as transfer losses, because of the connotation of transferring available energy to accomplish propulsion work. Strictly speaking, the energy is not lost, since the First Law forbids any energy loss. Rather, the degree of unavailability is increased by virtue of Second Law effects. One can visualize a portion of the available energy in the gas generator stream as being extracted by a turbine to drive a fan, which in turn transfers the energy to the bypass stream, which then passes through a duct to a nozzle to produce thrust. During each energy transfer process, a certain fraction is made unavailable, thus contributing to the transfer losses. Ultimately, the total accumulated unavailability appears as heat to be dissipated to the atmosphere in the exhaust stream.

Transfer losses, associated with each individual element of a turbofan engine, which independently contribute to the production of thrust, can be measured and evaluated in terms of the efficiencies or coefficients conventionally associated with that element. This would include the nozzle thrust coefficients, the inlet kinetic efficiency and fan adiabatic efficiency related to a turbofan's bypass flow, fan bypass duct pressure loss coefficient, and fan drive turbine adiabatic efficiency.

Applications

As a special case, assume that transfer losses are ignored. Further, assume that the primary and exhaust stream velocities are equal such that $V_e = V_d = V_j$. In this particular case

$$F_n \times V_0 = [\, W_e (V_j - V_0) + W_D (V_j - V_0)\,] \times V_0 / g \quad (9)$$

$$= (W_e + W_D)(V_j - V_0) \times V_0 / g$$

$$\text{Residual KE} = \tfrac{1}{2}g\,(W_e + W_D)(V_j - V_0)^2 \quad (10)$$

$$\eta_{\text{prop}} = \frac{(W_e + W_D)(V_j - V_0) \times V_0/g}{(W_e + W_D)(V_j - V_0) \times V_0/g + \tfrac{1}{2}g(W_e + W_D)(V_j - V_0)^2} \quad (11)$$

$$= V_0/[V_0 + \tfrac{1}{2}(Vj - v_0)] = V_0/[\tfrac{1}{2}Vj + \tfrac{1}{2}V_0] \quad (12)$$

$$= 2\,/\,(1 + Vj\,/V_0) \quad (13)$$

Expression (13) is the classic turbojet propulsive efficiency equation. It applies only in the assumed case of no transfer inefficiencies and, in the case of a turbofan, equal jet velocities. A more universal expression for propulsive efficiency would be

$$\eta_{\text{prop}} = (F_n \times V_0 / gJ)\,/\,(\Delta h_{\text{cycle}} \times W_e) \quad (14)$$

where

$F_n \times V_0$ = actual propulsive power

W_e = primary (gas generator) exhaust gas flow rate

Δh_{cycle} = total work ideally available from the thermodynamic cycle of the gas generator

where Δh_{cycle} usually must be determined from a fictional case. It represents the fraction of turbine work that powers the o.d. (bypass) portion of the fan only. It must be separated from the total low pressure turbine work, which also includes work required to power the i.d. portion of the fan and, in some cases, a low pressure compressor as well. The fan i.d. and low compressor contribute to the compression process of the thermodynamic cycle, and should not be included in the calculation of propulsive effects; Δh_{cycle} would, by definition, be computed for an ideal (isentropic) expansion process as illustrated in Fig. 3 for the conventional gas turbine cycle. Similar considerations would apply for reheat (afterburning), inter-cooled, regenerative, or other cycles.

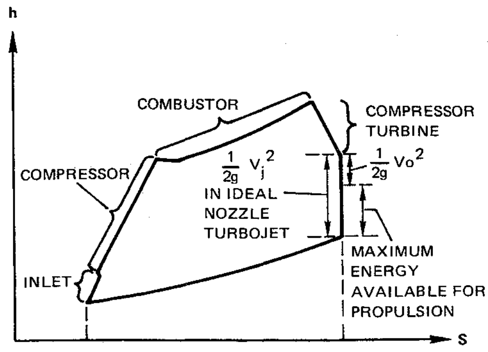

Fig. 3 Thermodynamic cycle *h-s* diagram.

It is interesting to note that, for a turbojet with an ideal exhaust nozzle

$$\Delta h_{\text{cycle}} = Q_A - Q_R \;= F_n \times V_0 / W_e + \textit{residual}\ \text{KE} \quad (15)$$

$$= (Vj - V_0) + V_0/g + \tfrac{1}{2}(V_j - V_0)^2 \quad (16)$$

$$= \tfrac{1}{2}g\,Vj^2 - \tfrac{1}{2}g\,V_0^2 \quad (17)$$

By inspecting the *h-s* diagram, it might appear offhand that the net cycle work for a turbojet would simply be equal to $\tfrac{1}{2}Vj^2$. However, by recognizing that the ram pressure ratio contributes to overall cycle pressure, one can see readily that the ram energy $\tfrac{1}{2}V_0^2$ must be provided by the cycle, analogous to 'compressor' work, and thus subtracted from the nozzle kinetic energy to determine the net cycle work. From another point of view, Eq. (17) simply states that the net cycle output work for a turbojet is identical to the change of kinetic energy of the air passing through the engine. Expansion of the energy balance to a turbofan having no transfer inefficiencies would generalize to the same result.

In the more general case of a turbofan, an expression equivalent to Eq. (17) for the available cycle energy can be developed to include terms that account for transfer inefficiencies. Although such development extends beyond the scope of the fundamental ideas presented in the foregoing discussion, it can be pointed out that applying the resulting general expression to the propulsive efficiency definition of Eq. (5) can lead to many useful theoretical results. For instance, a theoretical basis can be established for the optimization of fan pressure ratio, which must take place in the preliminary cycle selection process.[4] Another important example, the interrelationship of forward flight speed with bypass ratio (ratio of fan bypass and primary stream mass flow rotor), can be shown theoretically to depend strongly on transfer efficiency levels. Having at his disposal such theoretical bases for propulsive efficiency effects, the performance engineer can readily narrow down the initial selection of engine cycle design variables to achieve the optimum, most efficient powerplant design for any given application.

Summary

There is only one fundamental definition for propulsive efficiency which has universal application

$$\eta_{\text{prop}} = \frac{\textit{Actual propulsive power}}{\textit{Maximum available propulsive power}}$$

Maxmum available propulsive power is determined by the thermodynamic cycle of the gas generator. Cycle output relates to the sum total of the effects of cycle heat addition,cycle pressure ratio, and all of the inefficiencies (plus pressure losses) of the engine components associated with realizing heat addition and pressure ratio. The difference between actual and maximum propulsive power is manifested as waste energy which, by virtue of energy conservation, appears in the engine exhaust stream either as residual kinetic energy or as thermal energy. Ultimately, the wasted energy associated with thrust production is dissipated to the atmosphere along with the energy made unavailable by the thermodynamic cycle processes. In actuality, the cumulative entropy-producing processes are indistinguishable, appearing simply as a level of enthalpy of exhaust gases above ambient enthalpy. But, by arbitrarily dividing all of the waste energy into its constituent components, the performance engineer can isolate and analyze the sources of energy unavailabillty as they affect overall engine performance. For this purpose, the basic concept of propulsive efficiency can serve as a powerful analytical tool.

References

[1] Cohen, H., Rogers, G.F.C., and Saravanamitto, H.I.H. *Gas Turbine Theory*, 2d ed., Halsted Press, New York, 1972, pp. 59.

[2] Hill, P. G. and Peterson, E.R., *Mechanics and Thermodynamics of Propulsion*, Addison-Wesley, Cambridge, Massachusetts, 1965, pp. 143.

[3] Hesse, W.J., and Mumford, N.V.S., *Jet Propulsion for Aerospace Applications*, 2nd ed., Pitman, New York, 1964, pp. 39.

[4] Cohen, H. et al., *Jet Propulsion for Aerospace Applications*, 2nd ed., Pitman, New York, 1964, pp. 80-82.